C2 June 2013 (R) (MA)

Q1) $y = 2x + 3 + 8x^{-2}$

$\frac{dy}{dx} = 2 - 16x^{-3}$

at a stationary point, $\frac{dy}{dx} = 0$:

$2 - \frac{16}{x^3} = 0$

$2 = \frac{16}{x^3}$

$\therefore x^3 = 8$

$x = \sqrt[3]{8} = 2$ //

at $x = 2$, $y = 2(2) + 3 + 8(2)^{-2} = 9$ //

$\therefore$ the stationary point is $\boxed{(2, 9)}$

2a)

1.3
0.8572

b) $h = \frac{b-a}{n} = \frac{1.5-1}{5} = 0.1$ //

$\text{Area} \simeq \frac{1}{2} \times 0.1 \left[0.7071 + 0.9487 + 2(0.7591 + 0.8090 + 0.9037 + 0.8572)\right]$

$\simeq \boxed{0.416}$ (3d.p)

3) $\left(2-\frac{1}{2}x\right)^8 \simeq 2^8 + \binom{8}{1}\left(2^7\right)\left(-\frac{1}{2}x\right)^1$

$+ \binom{8}{2}(2)^6\left(-\frac{1}{2}x\right)^2 + \binom{8}{3}(2)^5\left(-\frac{1}{2}x\right)^3$

$\therefore \left(2-\frac{1}{2}x\right)^8 \simeq 256 + 1024\left(-\frac{1}{2}x\right) + 1792\left(\frac{x^2}{4}\right)$

$+ 1792\left(\frac{-x^3}{8}\right)$

$\left(2-\frac{1}{2}x\right)^8 \simeq 256 - 512x + 448x^2 - 224x^3$

4a) $f(x) = ax^3 - 11x^2 + bx + 4$

$f(x) \div (x-3) \rightarrow r = 55$

$\therefore f(3) = 55$

$\Rightarrow f(3) = 27a - 99 + 3b + 4 = 55$

$\Rightarrow \quad 27a + 3b = 150 \sim 1$

$f(x) \div (x+1) \rightarrow r = -9$

$\therefore f(-1) = -9$

$\Rightarrow f(-1) = -a - 11 + (-b) + 4 = -9$

$\Rightarrow \quad a + b = 2 \sim 2$

Solve 1 and 2 simultaneously:

from 2 : $b = 2 - a$. $\rightarrow$ 1 : $27a + 6 - 3a = 150$

$24a = 144$

$\therefore \boxed{a = 6}$

$\therefore b = 2 - 6 = \boxed{-4}$

b)

$$\begin{array}{r} 2x^2 - 5x + 2 \\ 3x + 2 \overline{\smash{)}\, 6x^3 - 11x^2 - 4x + 4} \\ 6x^3 + 4x^2 \qquad\qquad \\ \hline 0 \; - 15x^2 - 4x \quad\; \\ -15x^2 - 10x \quad\; \\ \hline 0 + 6x + 4 \\ 6x + 4 \\ \hline 0 \quad 0 \end{array}$$

//

$\therefore f(x) = (3x+2)(2x^2 - 5x + 2)$

but $2x^2 - 5x + 2 = (2x - 1)(x - 2)$

$\therefore f(x) = (3x+2)(2x-1)(x-2)$

5a) $4p, \quad 3p+15, \quad 5p+20$

$a \quad ar \quad ar^2$

$a = 4p$
$ar = 3p+15$.

$\frac{ar}{a} = \frac{3p+15}{4p} = r = \frac{3}{4} + \frac{15}{4p}$ //

$\therefore r^2 = \frac{(3p+15)^2}{16p^2} = \frac{9p^2+90p+225}{16p^2}$

and we know $ar^2 = 5p+20$.

$\therefore 4p\left(\frac{9p^2+90p+225}{16p^2}\right) = 5p+20$

$\frac{9p^2+90p+225}{4p} = 5p+20$

$9p^2+90p+225 = 4p(5p+20)$

$9p^2+90p+225 = 20p^2+80p$.

$\therefore 11p^2+80p-90p-225 = 0$

hence $\boxed{11p^2-10p-225=0}$

b) $11p^2 - 10p - 225 = 0$

$(11p + 45)(p - 5) = 0$

$11p + 45 = 0$
$\hookrightarrow p = -\frac{45}{11}$
reject $(p > 0)$.

$p - 5 = 0$
$\boxed{p = 5}$

c) from (a), $r = \frac{3p+15}{4p} = \frac{3(5)+15}{20} = \boxed{\frac{3}{2}}$

d) $a = 4p = 20$, $r = 1.5$ $\Big\}$ $S_{10} = \frac{20(1-(1.5^{10}))}{1-1.5} = 2266.6\ldots$

$= \boxed{2267}$
nearest int.

6a) $\log_3 x = a$

$\therefore \log_3(9x) = \log_3(9) + \log_3(x)$

$= \boxed{2 + a}$

b) $\log_3\left(\frac{x^5}{81}\right) = \log_3(x^5) - \log_3(81)$

$= 5\log_3(x) - 4$

$= \boxed{5a - 4}$

c) $\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$

from (a) and (b),

$(2+a) + (5a-4) = 3$

$6a - 2 = 3$

$6a = 5 \quad \therefore \boxed{a = \frac{5}{6}}$

and $\log_3 x = a$

$\therefore 3^a = x$

$\Rightarrow x = 3^{\frac{5}{6}} = \boxed{2.498}$

7a) $x^2 + 2x + 2 = 10.$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = 2, \quad x = -4. \quad \therefore \boxed{A(-4, 10) \quad B(2, 10)}$

b) $R = \int_A^B (y_2 - y_1)\,dx = \int_{-4}^{2} [8 - x^2 - 2x]\,dx$

It doesn't matter which function you select to be y_1 or y_2, as long as you take the modulus of your final answer.

$$R = \left[8x - \frac{x^3}{3} - x^2\right]_{-4}^{2} = \left[16 - \frac{8}{3} - 4\right] - \left[-32 + \frac{64}{3} - 16\right]$$

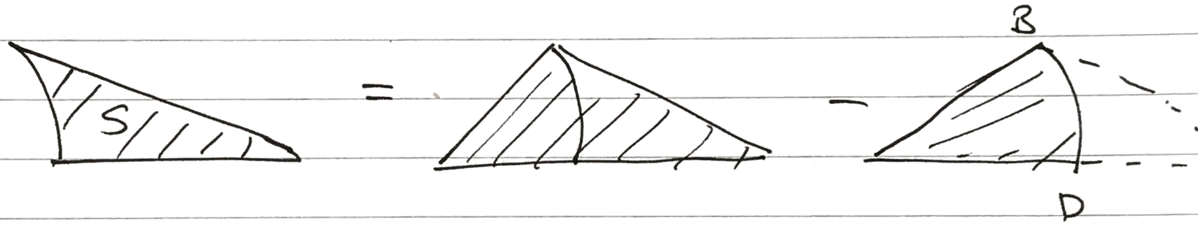
$$R = \frac{28}{3} + \frac{80}{3} = \boxed{36 \text{ units}^2}$$

8a) <u>cosine rule</u>: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$\cos(BAD) = \frac{13^2 + 7^2 - 10^2}{2(13)(7)} = \frac{59}{91}$$

$$\therefore \angle BAD = \cos^{-1}\left(\frac{59}{91}\right) = \boxed{0.865^c} \quad 3\text{d.p.}$$

b)



$$S = \text{Area}_{ABC} - \text{Area}_{ABD}$$

$$\therefore S = \tfrac{1}{2}(7)(13)\sin(0.865) - \tfrac{1}{2}(7^2)(0.865)$$

$$S = 34.6298\ldots - 21.1925\ldots = \underline{\underline{13.44\,m^2}}$$

$\therefore$ amount of grass seed needed $= 50 \times 13.44$
$= 671.865\,g$

$= \boxed{670\,g}$
nearest 10g

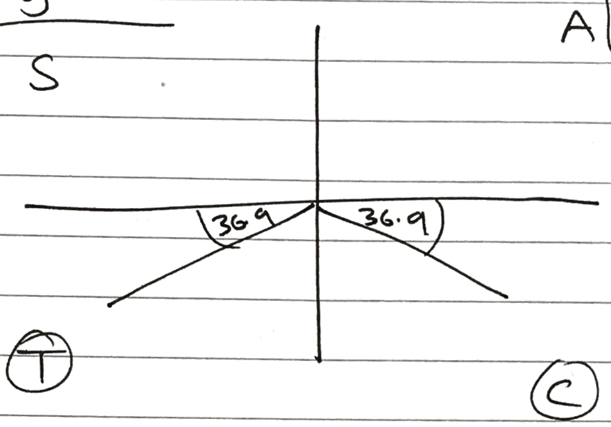
$= \cancel{\boxed{672\,g}}$
~~nearest g~~

9i) $\sin(2\theta - 30^\circ) = 0.4 - 1 = -0.6$

$\therefore \sin^{-1}(-0.6) = 2\theta - 30 = -36.87^\circ$.

Solving in: $-30^\circ \leq 2\theta - 30^\circ < 330$

By CAST:



$2\theta - 30 = 216.9^\circ, 323.1^\circ$

$2\theta = 246.9^\circ, 353.1^\circ$

$\boxed{\theta = 123.4^\circ, 176.6^\circ}$

* use 36.87° or better in your calculations to avoid inaccuracy to 1 d.p

ii) $9\cos^2 x - 11\cos x + 3(1 - \cos^2 x) = 0$

$9\cos^2 x - 11\cos x + 3 - 3\cos^2 x = 0$

$6\cos^2 x - 11\cos x + 3 = 0$

$(3\cos x - 1)(2\cos x - 3) = 0$

$3\cos x - 1 = 0$

$\cos x = \frac{1}{3}$

$2\cos x - 3 = 0$

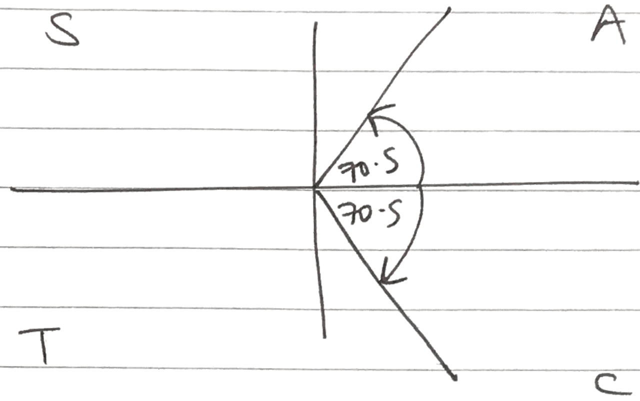
$\cos x = \frac{3}{2}$ × reject. no valid solutions ($\cos x \leq 1$).

$x = \cos^{-1}\left(\frac{1}{3}\right)$

$x = 70.53^\circ$

Solving in $0 \le x < 360$

CAST:



$x = 70 \cdot 5°, (360 - 70 \cdot 5°)$

$\Rightarrow \quad \boxed{x = 70 \cdot 5°, 289 \cdot 5°}$